6.2 ESTIMATION OF MODEL ERRORS IN THE LOCAL ENSEMBLE TRANSFORM KALMAN FILTER

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1. INTRODUCTION

Ensemble Kalman Filters (EnKF) have been shown to be more accurate than 3D-Var in data assimilation simulations under the assumption of a perfect model. However, in reality, the forecast model has deficiencies and does not represent the atmospheric behavior precisely due to lack of resolution, approximate parameterizations of subgrid scale physical processes, and numerical dispersion. For assimilation of real observations, into an imperfect model, it is not yet clear whether the EnKF will be competitive or better than the current operational 3D-Var data assimilation systems. Only recently have some EnKF schemes advanced from the perfect model scenario to real-world situations. Houtekamer et al (2005) showed that the quality of EnKF with perturbed observations was comparable to 3D-Var. The Ensemble Square-Root filter (EnSRF) was reported (Whitaker et al 2004) to outperform the NCEP 3D-Var in reconstructing the middle and lower tropospheric analysis in the Northern Hemisphere at T62/L28 resolution. Miyoshi (2005) also showed that EnKF is most advantageous over 3D-Var when the observing network is sparse and also that the advantage diminishes in the presence of model errors. Model errors have a stronger negative influence on the performance of the EnKF than on the 3D-Var because Kalman filtering algorithms rely strongly on the assumption of an unbiased model, an assumption which is not satisfied in practice. Therefore, accounting for systematic errors associated with model deficiencies has become an important issue for all data assimilation systems, and especially for EnKF.

The Local Ensemble Transform Kalman Filter (LETKF) (Hunt 2005) is a relatively new data assimilation scheme in the square root EnKF family, and is similar to the Local Ensemble Kalman Filter (Ott et al 2004) but faster. It has been implemented to assimilate simulated observations in the NCEP GFS model (Szunyogh et al. 2005), and recently in the NASA fvGCM model (Liu et al. 2005). The results are excellent for a perfect model scenario. In order to develop the LETKF into a competitive, operationally applicable data assimilation system, it is necessary to extend the application of the LETKF to more realistic weather forecast systems by accounting for systematic model errors.

The main goal in this work is to investigate techniques for treating model errors in the Ensemble Kalman Filter, building on previous work, and to develop a data assimilation system capable of assimilating real weather observations. Though we focus on the LETKF, the results may also be applicable to other Ensemble Kalman Filter methods. Recently, Baek et al (2005) extended the work of Dee and Da Silva (1998), hereafter referred as DDS, by using a high order bias estimate scheme based on the state augmentation correcting the model errors at each grid point, and also accounting for the cross-correlation of model state variables and bias. They successfully tested this approach with the Lorenz 40-variable model and showed the ability to correct forecast model errors. Miyoshi (2005) tested the high order DDS approach on the SPEEDY primitive equations model (without cross correlation terms) with the EnSRF assimilation scheme but found it to be unsuccessful. He then tried a low order correction approach developed by Danforth et al (2005) in which the model errors are expanded into low order EOFs, with very good results. In this work, by using the LETKF as the data assimilation method, we first test the Baek et al (2005) high order approach with cross-correlations, to determine whether the failure found by Miyoshi (2005) is due to the absence of cross-correlations. However the high order approach of correcting at each grid point is computationally very expensive. Thus we develop a low order approach in the LETKF, and compare both results, under the hypothesis that model errors can be represented by relatively few degrees of freedom and can thus be efficiently corrected.

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2. TECHNIQUES FOR TREATING MODEL ERRORS

2.1 Full dimensional augmented state method

Friedland (1969) proposed the augmented state method by augmenting the state vector by a model bias vector to estimate both the state and bias variables. Building on this idea, Dee and Da Silva (1998) developed a two-stage estimation algorithm in which the estimation procedures for the bias and the state are carried out successively. The bias is estimated on every model grid point by the observed-minus-forecast residuals $y^o - h(x^f)$

$$b^a = b^f - K_b[y^o - h(x^f) + h(b^f)] \quad (1)$$

$$K_b = P_{bb}^f H^T (HP_{bb}^f H^T + HP_{xx}^f H^T + R)^{-1} \quad (2)$$

where the matrix P_{xx}^f and P_{bb}^f are the forecast error covariance for the bias and the state variables, respectively. $h(\cdot)$ is the operator mapping the model state variables into observation space.

The analysis for the state variables is then obtained using the standard analysis procedure with the unbiased forecast state $\tilde{x}^f = x^f - b^a$.

Another augmented state method proposed by Baek et al (2005) is to obtain the optimal estimates of the state and bias variables simultaneously. They assume there are no bias observations but that the bias on all model grids can be updated from the state observations by the cross-correlation between the forecast state and bias through the cross covariance, while this cross-correlation is assumed to be zero in DDS.

The unbiased state forecast is calculated first as $\tilde{x}^f = x^f - b^f$, then is used in the augmented state analysis procedure:

$$\begin{pmatrix} x^a \\ b^a \end{pmatrix} = \begin{pmatrix} \tilde{x}^f \\ b^f \end{pmatrix} + K_{bx}[y^o - h(\tilde{x}^f)] \quad (3)$$

$$K_{bx} = \begin{pmatrix} P_{xx}^f & P_{xb}^f \\ P_{bx}^f & P_{bb}^f \end{pmatrix} \begin{pmatrix} H \\ 0 \end{pmatrix}^T (HP_{xx}^f H^T + R)^{-1} \quad (4)$$

Equations (3) and (4) implicate the bias is estimated by

$$\begin{aligned} b^a &= b^f + K_b[y^o - h(\tilde{x}^f)] \\ &= b^f + K_b[y^o - h(x^f) + h(b^f)] \end{aligned} \quad (5)$$

$$K_b = P_{bx}^f H^T (HP_{xx}^f H^T + R)^{-1} \quad (6)$$

The major difference between (1), (2) and (5), (6) exists in the weighting matrix K_b, the former uses the bias forecast error covariance P_{bb}^f, the latter uses the cross covariance P_{bx}^f between the bias and state variables.

2.2 Low order bias estimation scheme

Assume we have a simple nonlinear model such that

$$x_{n+1}^m = F_n^m(x_n^m) = a^m + b^m x_n^m + c^m x_n^{m2} + \ldots \quad (7)$$

where *n* is the time step, x_n represents the anomaly of the state with respect to climatology, and $a^m, b^m, c^m, \ldots$ are coefficients of state-independent terms, and of terms that vary linearly, quadratically or with higher order with the state anomaly. These model parameters may be a function of time or space, containing, for example, a diurnal and an annual cycle. The model approximates the evolution of a "true" system:

$$x_{n+1}^t = F_n^t(x_n^t) = a^t + b^t x_n^t + c^t x_n^{t\,2} + \ldots \quad (8)$$

Here the super-indices *m* and *t* refer to model and truth respectively. We define the state error (with the sign of a correction) as

$\delta x_n = x_n^t - x_n^m$, so that

$$\delta x_{n+1} = F_n^t(x_n^t) - F_n^m(x_n^m) \quad (9)$$

We measure noisy observations of the truth

$$\bar{x}_n^t = x_n^t + \delta x_n^t$$

Then

$$\begin{aligned} \delta x_{n+1} &= [\delta a + \delta b x_n^m + \delta c x_n^{m2} + \ldots] \\ &+ [(b^m + 2c^m x_n^{m2} + \ldots)\delta x_n] + o(\delta F)o(\delta x_n) \end{aligned} \quad (10)$$

Here δF represents $\delta a = a^t - a^m$, $\delta b = b^t - b^m$, $\delta c = c^t - c^m, \ldots$, the model parameter errors. The first bracket on the RHS of (10) represents error due to model deficiencies, i.e., the source of "external" error growth, and the second is the linear tangent model or propagator of the error that is the source of "internal" error growth related to instabilities. We will neglect the last terms that are proportional to the product of parameter errors and state errors.

If the true state was exactly known ($\delta x_{n+1}^t = 0$), we could use the model (7) and initial conditions x_n^t and compute exactly the model error at every time step. However, in practice we only have noisy observations of the truth. If we assume that the observational error δx_n^t and state error δx_n have zero mean and their standard deviation is small compared to the typical size of the model state anomaly, then we can measure equation (10) over many cases and obtain

$\delta x_{n+1} = \left[\delta a + \delta b x_n^m + \delta c x_n^{m2} + ... \right]$ + random errors

If we collect enough cases, we can obtain the systematic errors due to model deficiencies by linear regression where the predictor is x_n^m.

Danforth et al (2005) have performed this procedure with the SPEEDY model using 5 years of reanalysis as training for the linear regression. Like Leith (1978) and DelSole and Hou (1999), they retained only the state independent and the linear terms. In the multiple linear regression, they obtained the seasonally dependent model bias and the diurnal errors that dominate the EOFs of the state independent error corresponding to δa. The linear component corresponding to δb was obtained using SVD of the covariance of the coupled model state anomalies and corresponding measured errors with respect to the reanalysis. This required spatial localization of the covariance matrix in order to reduce sampling problems. The model bias δa and the leading EOFs of the state independent and state dependent errors explain most of the model error and can be used as a low order model correction.

In this study we plan to take advantage of these independently derived fields and assume that during the data assimilation the actual errors are proportional to them, with amplitudes that are determined as part of the LETKF cycle.

$$b = T\,\beta \quad (11)$$

Here T denotes the pre-computed base fields with dimension n×k, where each column denotes base fields, including the seasonal bias, the diurnal errors corresponding to` δa, and the state dependent errors. The time-dependent amplitudes of these base fields are represented by the vector β (with dimension k) and can be estimated in the analysis cycle. Since the dimension of β is much lower than the model dimension n, estimating β is far less costly than estimating b in the analysis cycle.

3. IMPLEMENTATION ON THE LETKF IN THE PRESENCE OF THE MODEL ERRORS

3.1 The SPEEDY model

The SPEEDY model (Molteni 2003) is a recently developed atmospheric general circulation model (AGCM) with simplified physical parameterization schemes that are computationally efficient, but that maintain the basic characteristics of a state-of-the-art AGCM with complex physics. It has a spectral primitive-equation dynamics and triangular truncation T30 at 7 sigma levels.

3.2 LETKF data assimilation scheme

The Local Ensemble Transform Kalman Filter (LETKF) is chosen for this model error estimation study since it belongs to the sequential data assimilation family whose performance is sensitive to model bias.

LETKF is an ensemble square-root filter in which the observations are assimilated to update only the ensemble mean (shown in equation (12)) while the ensemble perturbations are updated by transforming the forecast perturbations through a transform matrix (equation (13)) introduced by Bishop et al (2001). The basic formulas used in the LETKF (Hunt 2005) are given by

$$\bar{x}^a = \bar{x}^b + X^b \tilde{P}^a (HX^b)^T R^{-1} [y^o - h(\bar{x}^b)] \quad (12)$$

$$X^a = X^b [(k-1)\tilde{P}^a]^{1/2} \quad (13)$$

Here X^a, X^b are the analysis and forecast ensemble perturbations, respectively. The transform matrix $\tilde{P}^{a1/2}$ is the square-root of matrix $(k-1)\tilde{P}^a$ where $\tilde{P}^a$ is given by

$$\tilde{P}^a = \left[(k-1)I + (HX^b)^T R^{-1} (HX^b)\right]^{-1} \quad (14)$$

with the dimension of k by k, where k is the ensemble size, which is generally much smaller than both the dimension of the model and the number of observations. Thus, the LETKF performs the analysis in the space spanned by the forecast ensemble members, which greatly reduces the computational cost. Furthermore, since the analysis is computed independently at each grid point, the LETKF computation can be performed in parallel.

3.3 Observations

The observations are obtained by adding the zero mean normal distributed noise to the NCEP/NCAR reanalysis (NNR) fields (Kalnay et al 1996), which are then an indirect estimate of the state of the evolving atmosphere. With respect to these observations, the SPEEDY model has significant model errors. The observations are available on the model grid at every 4 grid points. In each analysis cycle (6hour), the LETKF is combined with one of the bias estimate methods to correct the forecast errors due to inaccurate initial conditions and model errors.

4. PRELIMINARY RESULTS

4.1 Evidence of model bias

First we investigate the SPEEDY model bias against the NNR. The SPEEY model is evolved from the NNR every 6 hours. Figure 1 shows the differences between the SPEEDY 6hr forecasts and the NNR verified at the same time, averaged over two months in the period from January 1, 1982 to February 28, 1982, for the zonal wind and height at 500 hPa. The largest model bias of the u-wind can be seen in the polar regions. Orographic effect is a major originator for the systematic errors in the height field.

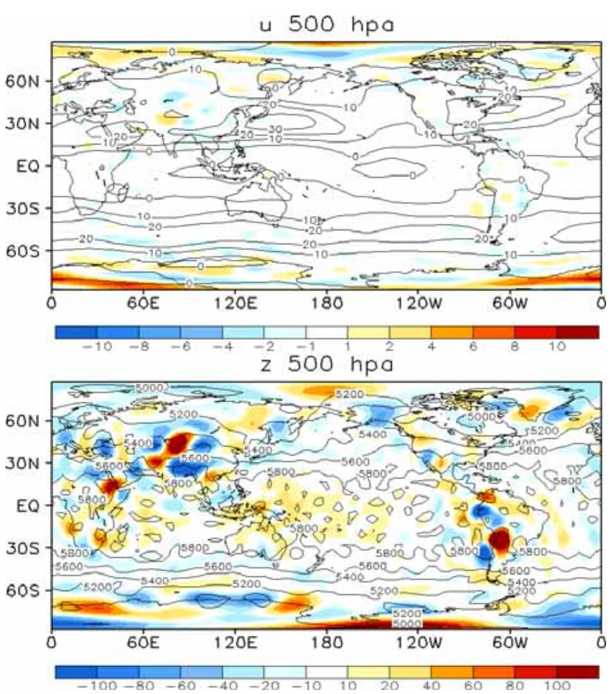

FIG. 1 The SPEEDY 6-hour model bias of u-wind (unit: m/s) and height (unit: m) at 500hPa against NNR reanalysis fields. The time averaged NNR over the same period and at the same level is shown by contours.

4.2 Effects of model errors on the LETKF

To investigate the effects of model errors, we perform the LETKF using “realistic observations” from NNR and “simulated observations” obtained from a SPEEDY model “nature run” where no model errors exist. Figure 2 shows the strong negative influence of the model errors on the performance of the LETKF. In the presence of model errors, the analysis is much worse than in the perfect model scenario for u-wind, height, and other model variables (not shown) at all pressures levels. Therefore, it is an important issue for LETKF to estimate and correct the model errors.

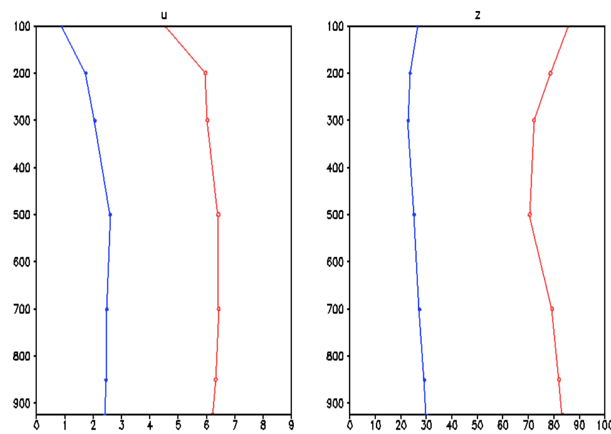

FIG.2 Global mean analysis RMS error at all pressure levels averaged over the second month (Feb 1982) after the initial transient behavior with LETKF in the perfect model scenario (blue line) and in the presence of the model errors (red line) for u-wind (left) and height (right).

4.3 *Constant mean bias correction*

Following Miyoshi (2005), we simplify the low order estimation scheme by correcting only the time-averaged fields. In this way, T in equation (11) has just one column, given by the mean bias over the two months period. β is fixed to 1. Figure 3 illustrates the success of this simple bias correction. Comparing the analysis RMSE at 500hPa with (green line) and without (red line) the mean bias correction, we can see clearly that subtracting the mean bias largely reduced the analysis errors. This improvement is also substantial at other pressure levels especially at the lower levels (Figure 4).

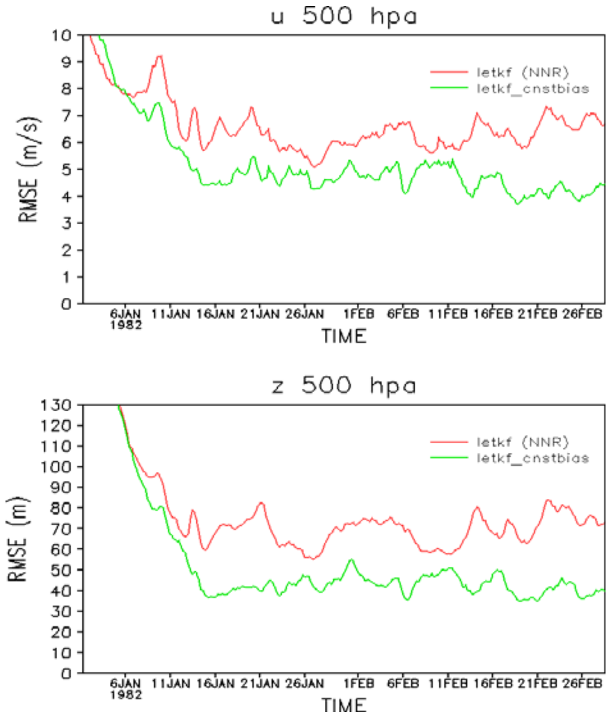

FIG.3 Time evolution of the global mean analysis errors of the u-wind (top) and height (bottom) fields at 500hPa in the presence of the model errors (red line) and in the case of mean bias correction.

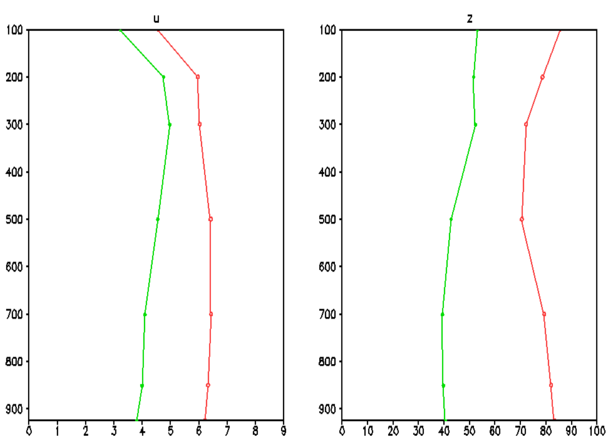

FIG.4 Global mean analysis RMS at all pressure levels averaged over the second month (Feb 1982) beyond the spin-up time in the presence of the model errors (red line) and in the case of mean bias correction (green line) for u-wind (left) and height (right).

5. DISCUSSION AND FUTURE WORK

The full dimensional bias estimation method has not been tested so far. We will test the augmented state method in our future work. However this high order approach of correcting at each grid point is computationally expensive and can lead to very slow convergence of the LETKF because it doubles or triples the number of unknowns to be determined. According to Baek et al (2005), in order to obtain the optimal analysis, the number of the ensemble members also needs to be doubled, making the computationally cost even higher. In contrast, the low order approach increases the size of the model state by a very small percentage, the expectation is that it will be much more efficient than an augmentation with the full model error field.

Our results have shown that simply subtracting the constant mean bias from the background fields at every analysis cycle has a significant positive impact on the LETKF. In our future work, the time-variant amplitude of this mean bias field will be estimated by using the low order method. We believe this will give us a better result than the constant mean bias subtraction. Moreover, we will correct the diurnal bias and then the state-dependent bias. Retaining each component of the bias estimate, we should be able to get a good estimate of the true model errors. If successful, this approach could not only improve the performance of the LETKF, but also of the forecast, as well as providing information on model errors useful for diagnostic purposes.

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